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RANDOM HEAT TRANSFER IN FLAT CHANNELS WITH TIMEWISE VARIATION OF AMBIENT TEMPERATURE

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Abstract—This paper discusses an analytical investigation for the random entrance region heat transfer of a fluid in a parallel plate channel. The environment surrounding the channel experiences randomly varying temperature oscillations. The mathematical analysis utilized is centered on a lumped formulation in the transversal direction of the channel. Computed results provide the statistical parameters such as the mean temperature distribution and the variance distribution for a group of heat-transfer situations. Estimates for the random behavior of the fluid temperature can be deducted from a set of figures involving several examples.

NOMENCLATURE

- b, half-width of channel;
- f, function defined in equation (16a);
- g, function defined in equation (16a);
- H, overall heat-transfer coefficient;
- k, fluid thermal conductivity;
- N, convective parameter, Hb/k;
- t, time;
- T, fluid temperature;
- T_a, ambient temperature;
- T_e, entrance temperature at $x = 0$ and initial temperature in the channel;
- $\bar{u}$, mean fluid velocity;
- U, dimensionless velocity parameter, $\bar{u}b/\alpha$;
- v, covariance of fluid temperature in equation (28);
- V, function defined in equation (29);
- W, variance of the ambient temperature;
- x, coordinate.

Greek symbols

- α, fluid thermal diffusivity;
- δ, Dirac's delta function;
- η, dimensionless coordinate, x/b;
- ξ, coordinate for the covariance in equation (28);
- θ, dimensionless fluid temperature, T/T_e;
- $\hat{\theta}$, mean fluid temperature;
- θ_a, dimensionless ambient temperature, T_a/T_e;
- $\hat{\theta}_a$, mean ambient temperature;
- $\tilde{\theta}_a$, temperature deviation, $\theta_a - \hat{\theta}_a$;
- τ, dimensionless time, $\alpha t/b^2$.

INTRODUCTION

THE PROBLEM of unsteady thermal convection in internal flows has been widely studied as evidenced in [1–13] and the references cited therein also. Conventionally, these publications deal with the heat-transfer analysis based on the use of deterministic functions of temperature. Naturally, this is the case when the effects of random velocity and/or random temperature fluctuations are virtually negligible when compared with mean values, and therefore local variances are relatively small. The solutions to this class of problems are characterized by a single parameter, the mean temperature. However, the randomness of temperature becomes increasingly significant when the velocity or temperature fluctuations are large enough, and ultimately may attain values wherein the contribution of these fluctuations can no longer be ignored. Undoubtedly, in order to get confidence limits for this class of problems, the mean temperature has to be linked to its corresponding variance.

One paper by Perlmutter [14] has been reported in the literature dealing with the heat-transfer estimate of a fluid in a parallel plate channel having randomly changing velocities. The solution method is centered on the application of model sampling and the use of Monte Carlo techniques.

In most heat-transfer studies, fluid temperatures possess a smooth behavior which can be adequately described employing deterministic functions. This specification becomes less realistic and almost impossible to impose as temperature variances become large. This may be the situation for calculating the heat loss from a channel wherein the random character is due to: (a) random velocities [14]; (b) random concentrations [15]; or (c) random thermal conditions. The latter can be caused by exposing the channel to an environment whose temperature changes abruptly with time due perhaps to unexpected weather conditions or any other external phenomenon.

The purpose of this paper is to describe a theoretical investigation concerned with the influence of randomly varying ambient temperatures on the heat-transfer performance of a fluid flow through a parallel plate channel. In the analysis, the governing

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energy equation is simplified by way of using a lumped formulation in the nonaxial direction. Consequently, the random contribution of the environment temperature appears in the energy equation characterizing the thermal process.

STATEMENT OF THE PROBLEM

This problem is concerned with the heat transfer in a hydrodynamically developed fluid flow in a flat conduit. The essential feature of the study deals with the calculation of the time-dependent fluid temperature in the axial direction. This dependency is caused by a random fluctuation of the ambient temperature surrounding the conduit, while the remaining conditions are maintained constant. A lumped formulation in the transverse direction, leads to an energy equation written in dimensionless form as follows

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\eta^2}-U\frac{\partial\theta}{\partial\eta}-N[\theta-\theta_a(\tau)] \quad (1)$$

where $\theta_a(\tau)$ denotes the random variation of the ambient temperature. This equation is based on the assumptions of constant fluid properties and negligible frictional dissipation of energy. The participating variables and parameters are chosen according to

$$\eta=\frac{x}{b} \qquad \tau=\frac{\alpha t}{b^2} \qquad \theta=\frac{T}{T_e}$$

$$U=\frac{\bar{u}b}{\alpha} \qquad N=\frac{Hb}{k}$$

while the rest of the variables appear defined in the Nomenclature.

It should be noted that equation (1) contains the axial conduction term. However, the intention of the present paper is not related to the exploration of the effects of conduction in the longitudinal direction. This approach is done with the sole purpose of facilitating the mathematical analysis to be presented in the next section. Generally, the real contribution of axial conduction is reflected only when equation (1) is applied to a domain that accommodates the distorted temperature profile upstream of the origin. The situation involving longitudinal conduction is insignificant when $P_e>50$ for constant wall temperature in a circular tube as suggested in [16, 17]. This value is even lower and depends upon the cooling level for conditions involving external convection at the tube walls [18].

The boundary and initial thermal conditions associated to equation (1) are expressed as follows

$$\begin{aligned}\theta(0,\tau)&=1\\ \theta(\infty,\tau)&=\text{finite}\end{aligned} \quad (2)$$

$$\theta(\eta,0)=1. \quad (3)$$

SOLUTION PROCEDURE

The temperature field $\theta(\eta,\tau)$ will be obtained from the statistics of the problem. It is based on the hypothesis that the ambient temperature $\theta_a(\tau)$ is a stationary normal process with mean and covariance given by the relations (see Appendix 1)

$$E[\theta_a(\tau)]=\hat{\theta}_a \quad (4)$$

and

$$E[\tilde{\theta}_a(\tau)\tilde{\theta}_a(\tau')]=W\delta(\tau-\tau') \quad (5)$$

respectively. In these expressions, $\hat{\theta}_a$ is the mean ambient temperature, $\tilde{\theta}_a$ is the temperature deviation $\theta_a-\hat{\theta}_a$, W is the variance of the ambient temperature, δ is a Dirac's delta function and finally E denotes the expectation.

We need to calculate the first and second moments (mean and covariance) only, because the governing system is linear and therefore $\theta(\eta,\tau)$ becomes a normal process. In terms of symbols, $\hat{\theta}(\eta,\tau)$ is used for the mean at η and additionally $v(\xi,\eta,\tau)$ is used for the covariance at any two points ξ and η, respectively.

First, the mean will be evaluated by solving the equation

$$\frac{\partial\hat{\theta}}{\partial\tau}=\frac{\partial^2\hat{\theta}}{\partial\eta^2}-U\frac{\partial\hat{\theta}}{\partial\eta}-N(\hat{\theta}-\hat{\theta}_a) \quad (6)$$

subject to the conditions

$$\begin{aligned}\hat{\theta}(0,\tau)&=1\\ \hat{\theta}(\infty,\tau)&=\text{finite}\end{aligned} \quad (7)$$

$$\hat{\theta}(\eta,0)=1. \quad (8)$$

The solution of equation (6) is divided in two parts via the superposition principle, i.e.

$$\hat{\theta}(\eta,\tau)=\hat{\theta}_1(\eta)+\hat{\theta}_2(\eta,\tau) \quad (9)$$

such that the steady state component $\hat{\theta}_1(\eta)$ is obtained from

$$\frac{d^2\hat{\theta}_1}{d\eta^2}-U\frac{d\hat{\theta}_1}{d\eta}-N(\hat{\theta}_1-\hat{\theta}_a)=0 \quad (10)$$

$$\hat{\theta}_1(0)=1 \quad (11)$$

and the unsteady state component $\hat{\theta}_2(\eta,\tau)$ is calculated from

$$\frac{\partial\hat{\theta}_2}{\partial\tau}=\frac{\partial^2\hat{\theta}_2}{\partial\eta^2}-U\frac{\partial\hat{\theta}_2}{\partial\eta}-N\hat{\theta}_2 \quad (12)$$

$$\hat{\theta}_2(0,\tau)=0 \quad (13)$$

$$\hat{\theta}_2(\eta,0)=1-\hat{\theta}_1(\eta). \quad (14)$$

At this point, it is easily seen that the expression

$$\hat{\theta}_1=\hat{\theta}_a+(1-\hat{\theta}_a)\exp\left\{\left[\frac{U-(U^2+4N)^{1/2}}{2}\right]\eta\right\} \quad (15)$$

satisfies equations (10) and (11).

The solution $\hat{\theta}_2(\eta,\tau)$ can be readily determined by introducing the transformation

$$\hat{\Theta}_2=f(\tau)g(\eta)\hat{\theta}_2(\eta,\tau) \quad (16)$$

into equation (12), where the functions are designated by

$$f(\tau) = \exp\left[\left(N + \frac{U^2}{4}\right)\tau\right] \quad g(\eta) = \exp\left(-\frac{U}{2}\eta\right). \quad (16a)$$

As a result, the new equation in terms of $\hat{\Theta}_2$ is given by

$$\frac{\partial \hat{\Theta}_2}{\partial \tau} = \frac{\partial^2 \hat{\Theta}_2}{\partial \eta^2} \quad (17)$$

in conjunction to the conditions

$$\hat{\Theta}_2(0, \tau) = 0 \quad (18)$$

$$\hat{\Theta}_2(\eta, 0) = g(\eta)[1 - \hat{\theta}_1(\eta)]. \quad (19)$$

Hence, the solution of this system can be written immediately from [19, 20] as follows

$$\hat{\Theta}_2(\eta, \tau) = \frac{1}{2(\pi\tau)^{1/2}} \int_0^\infty \hat{\Theta}_2(\eta', 0)\left\{\exp\left[-\frac{(\eta-\eta')^2}{4\tau}\right] - \exp\left[-\frac{(\eta+\eta')^2}{4\tau}\right]\right\} d\eta'. \quad (20)$$

But, since

$$\hat{\Theta}_2(\eta, 0) = (1 - \hat{\theta}_a)\left(1 - \exp\left\{\left[\frac{U - (U^2+4N)^{1/2}}{2}\right]\eta\right\}\right)\exp\left(\frac{U}{2}\eta\right)$$

then, equation (20) can be arranged as

$$\hat{\Theta}_2(\eta, \tau) = \frac{1-\hat{\theta}_a}{2(\pi\tau)^{1/2}} \int_0^\infty \left\{\exp\left(-\frac{U\eta'}{2}\right) - \exp\left[-\frac{(U^2+4N)^{1/2}}{2}\eta'\right]\right\}\left\{\exp\left[-\frac{(\eta-\eta')^2}{4\tau}\right] - \exp\left[-\frac{(\eta+\eta')^2}{4\tau}\right]\right\} d\eta'. \quad (21)$$

Once the integration process is performed, the previous equation can be condensed in the following manner

$$\begin{aligned}\hat{\Theta}_2(\eta, \tau) = \frac{1-\hat{\theta}_a}{2}\Bigg(&\exp\left[\frac{U^2}{4}\tau - \frac{U}{2}\eta\right]\left\{1 + \operatorname{erf}\left[\frac{\eta - U\tau}{2(\tau)^{1/2}}\right]\right\} - \exp\left[\frac{U^2}{4}\tau + \frac{U}{2}\eta\right]\left\{1 - \operatorname{erf}\left[\frac{\eta + U\tau}{2(\tau)^{1/2}}\right]\right\}\\ &- \exp\left[\left(\frac{U^2}{4} + N\right)\tau - \frac{(U^2+4N)^{1/2}}{2}\eta\right]\left\{1 + \operatorname{erf}\left[\frac{\eta - (U^2+4N)^{1/2}\tau}{2(\tau)^{1/2}}\right]\right\}\\ &+ \exp\left[\left(\frac{U^2}{4} + N\right)\tau + \frac{(U^2+4N)^{1/2}}{2}\eta\right]\left\{1 - \operatorname{erf}\left[\frac{\eta + (U^2+4N)^{1/2}\tau}{2(\tau)^{1/2}}\right]\right\}\Bigg)\end{aligned} \quad (22)$$

where

$$\operatorname{erf}(x) = \frac{2}{(\pi)^{1/2}} \int_0^x \exp(-x^2)\,dx. \quad (23)$$

Therefore, the original variables are recovered using the inverse transformation expressed by equation (16). This, of course, enables conversion of equation (22) into

$$\begin{aligned}\hat{\theta}_2(\eta, \tau) = \frac{1-\hat{\theta}_a}{2}\Bigg(&\exp(-N\tau)\left\{1 + \operatorname{erf}\left[\frac{\eta - U\tau}{2(\tau)^{1/2}}\right]\right\}\\ &- \exp(-N\tau + U\eta)\left\{1 - \operatorname{erf}\left[\frac{\eta + U\tau}{2(\tau)^{1/2}}\right]\right\}\\ &- \exp\left\{\left[\frac{U - (U^2+4N)^{1/2}}{2}\right]\eta\right\}\left\{1 + \operatorname{erf}\left[\frac{\eta - (U^2+4N)^{1/2}\tau}{2(\tau)^{1/2}}\right]\right\}\\ &+ \exp\left\{\left[\frac{U + (U^2+4N)^{1/2}}{2}\right]\eta\right\}\left\{1 - \operatorname{erf}\left[\frac{\eta + (U^2+4N)^{1/2}\tau}{2(\tau)^{1/2}}\right]\right\}\Bigg).\end{aligned} \quad (24)$$

Second, the covariance $v(\xi, \eta, \tau)$ constitutes an integral part of the analysis and will be calculated by solving the equation

$$\frac{\partial v}{\partial \tau} = \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} - U\frac{\partial}{\partial \xi} - U\frac{\partial}{\partial \eta} - 2N\right)v + N^2 W \quad (25)$$

whose detailed derivation is presented in Appendix 2. The initial and boundary conditions imposed on this equation are written as

$$v(\xi, \eta, 0) = 0 \quad (26)$$

$$v(0, \eta, \tau) = 0 \quad \text{or} \quad v(\xi, 0, \tau) = 0 \quad (27)$$

where the covariance is described by the relation

$$v(\xi,\eta,\tau) = E\{[\theta(\xi,\tau)-\hat{\theta}(\xi,\tau)][\theta(\eta,\tau)-\hat{\theta}(\eta,\tau)]\} \tag{28}$$

or

$$v(\xi,\eta,\tau) = E[\tilde{\theta}(\xi,\tau)\tilde{\theta}(\eta,\tau)].$$

Basically, the solution of equation (25) will be attempted by invoking the following transformation:

$$V(\xi,\eta,\tau) = f^2(\tau)g(\xi)g(\eta)v(\xi,\eta,\tau). \tag{29}$$

Accordingly, equation (29) is converted to

$$\frac{\partial V}{\partial \tau} = \frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \eta^2} + N^2 W f^2(\tau) g(\xi) g(\eta) \tag{30}$$

together with the initial and boundary conditions

$$V(\xi,\eta,0) = 0 \tag{31}$$

$$V(0,\eta,\tau) = 0 \quad \text{or} \quad V(\xi,0,\tau) = 0. \tag{32}$$

The function $V(\xi,\eta,\tau)$, satisfying the system of equations (30)–(32), may be obtained directly from [19] and written as follows:

$$V(\xi,\eta,\tau) = N^2 W \int_0^\tau \mathrm{d}\tau' \int_0^\infty \mathrm{d}\xi' \int_0^\infty \frac{1}{4\pi(\tau-\tau')}\left\{\exp\left[-\frac{(\xi-\xi')^2}{4(\tau-\tau')}\right] - \exp\left[-\frac{(\xi+\xi')^2}{4(\tau-\tau')}\right]\right\}$$
$$\times\left\{\exp\left[-\frac{(\eta-\eta')^2}{4(\tau-\tau')}\right] - \exp\left[-\frac{(\eta+\eta')^2}{4(\tau-\tau')}\right]\right\}\exp\left[2\left(N+\frac{U^2}{4}\right)\tau'\right]\exp\left[-\frac{U}{2}(\xi'+\eta')\right]\mathrm{d}\eta'. \tag{33}$$

Hence, carrying out the integration process for both indefinite integrals, equation (33) may be rearranged as

$$V(\xi,\eta,\tau) = \frac{N^2 W}{4}\int_0^\tau \left\{\operatorname{erfc}\left[-\frac{\xi-U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right]\exp\left(-\frac{U}{2}\xi\right) - \operatorname{erfc}\left[\frac{\xi+U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right]\exp\left(\frac{U}{2}\xi\right)\right\}$$
$$\times\left\{\operatorname{erfc}\left[-\frac{\eta-U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right]\exp\left(-\frac{U}{2}\eta\right) - \operatorname{erfc}\left[\frac{\eta+U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right]\exp\left(\frac{U}{2}\eta\right)\right\}$$
$$\times\exp\left[\frac{U^2(\tau-\tau')}{2} + 2\left(N+\frac{U^2}{4}\right)\tau'\right]\mathrm{d}\tau' \tag{34}$$

where $\operatorname{erfc}(x) = 1-\operatorname{erf}(x)$.

As a final step, the combination of equations (16a), (29) and (34) provides the complete expression for the covariance:

$$v(\xi,\eta,\tau) = \frac{N^2 W}{4}\int_0^\tau \left\{\operatorname{erfc}\left[-\frac{\xi-U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right] - \operatorname{erfc}\left[\frac{\xi+U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right]\exp(U\xi)\right\}$$
$$\times\left\{\operatorname{erfc}\left[-\frac{\eta-U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right] - \operatorname{erfc}\left[\frac{\eta+U(\tau-\tau')}{2(\tau-\tau')^{1/2}}\right]\exp(U\eta)\right\}\exp[-2N(\tau-\tau')]\,\mathrm{d}\tau'. \tag{35}$$

NUMERICAL RESULTS

This study involves four independent parameters: $\hat{\theta}_a$, the mean of the ambient temperature; W, the variance of the ambient temperature; N, the convective parameter and U, the dimensionless velocity parameter. Because of the range of variation of each parameter, computations are made for a limited number of cases; i.e. $\hat{\theta}_a = 0.2$ and 0.5, $W = 0.4$, $N = 10$, 100 and 500, and $U = 100$. The rationale for the use of a large value of U is that the complete role of the axial conduction must not be covered by the present solution. Additionally, the numerical values assigned to the governing parameters are expected to represent a wide spectrum of physical situations, where the emphasis is concentrated on those intimately associated to the random nature of the problem, such as $\hat{\theta}_a$, W and N.

The average temperature profile $\hat{\theta}(\eta,\tau)$ is calculated from the resulting combination of equations (9), (15) and (24). Likewise, the companion physical quantity, the variance $v(\eta,\eta,\tau)$ of the temperature

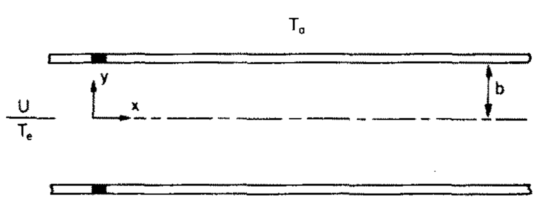

FIG 1. Coordinate system for the problem.

profile is computed from equation (34) for the condition $\xi = \eta$. In this last equation, the integration procedure over time is carried out by means of the Simpson's rule.

to the environment. It should be stressed that v shows a minor increasing behavior with both position and time, and, therefore, the heat-transfer mechanism is basically deterministic. At this level of

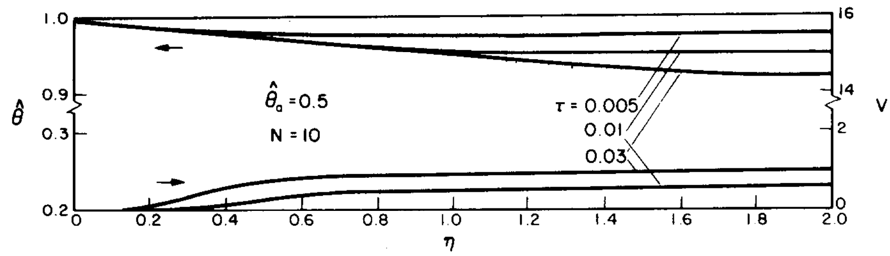

FIG. 2. Mean and variance of the temperature profile for $N = 10$ and $\hat{\theta}_a = 0.5$.

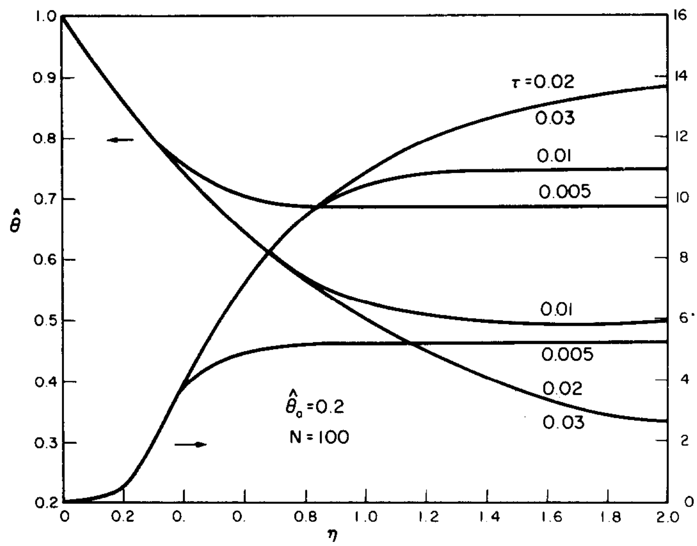

FIG. 3. Mean and variance of the temperature profile for $N = 100$ and $\hat{\theta}_a = 0.2$.

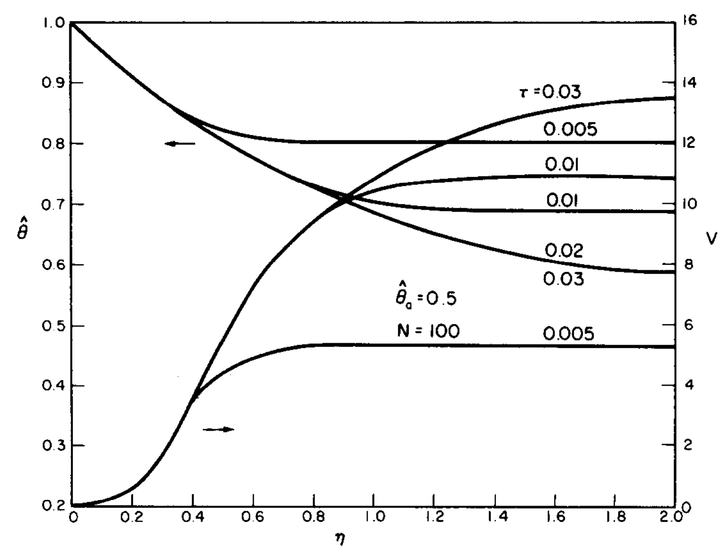

FIG. 4. Mean and variance of the temperature profile for $N = 100$ and $\hat{\theta}_a = 0.5$.

Figure 2 is an example of a typical solution for low cooling to the environment represented by $N = 10$ and $\hat{\theta}_a = 0.5$. Both quantities $\hat{\theta}$ and v do not change much with position due to the small heat exchange cooling, the temperature fluctuations around the mean may be considered as a secondary effect.

The variation of the average temperature and its variance with the axial location η and time τ is illustrated in Figs. 3 and 4 for $N = 100$ and $\hat{\theta}_a = 0.2$ and 0.5 respectively. Here, due to a higher cooling, local temperature drops are more pronounced. These figures reflect that for equal values of η and τ, the local temperatures tend to decrease as $\hat{\theta}_a$ increases. Conversely, variances are not altered by $\hat{\theta}_a$ and remain constant. For a fixed position, these variances tend to increase with time until they reach a steady-state value showing that temperature fluctuations are significant for $N = 100$. Alternatively, this behavior may be explained in a more concise manner, stating that as time progresses, a band representing the confidence limits for temperature becomes wider.

Finally, the influence of a very high cooling process characterized by $N = 500$ is drawn in Figs. 5 and 6 for the same ambient temperatures of $\hat{\theta}_a = 0.2$ and 0.5. These figures correspond to a limiting case and depict a sudden drop of the average temperature near the channel origin and eventually reach the average equilibrium temperature $\hat{\theta}_a$. In addition to this, the associated numerical values of the variances increase abruptly near the entrance as a function of time, and exhibit the same pattern regardless of the value of $\hat{\theta}_a$.

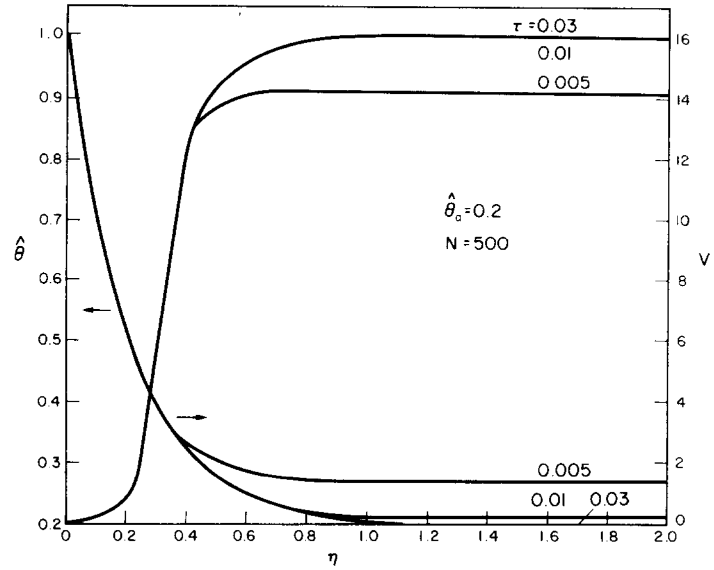

FIG. 5. Mean and variance of the temperature profile for $N = 500$ and $\hat{\theta}_a = 0.2$.

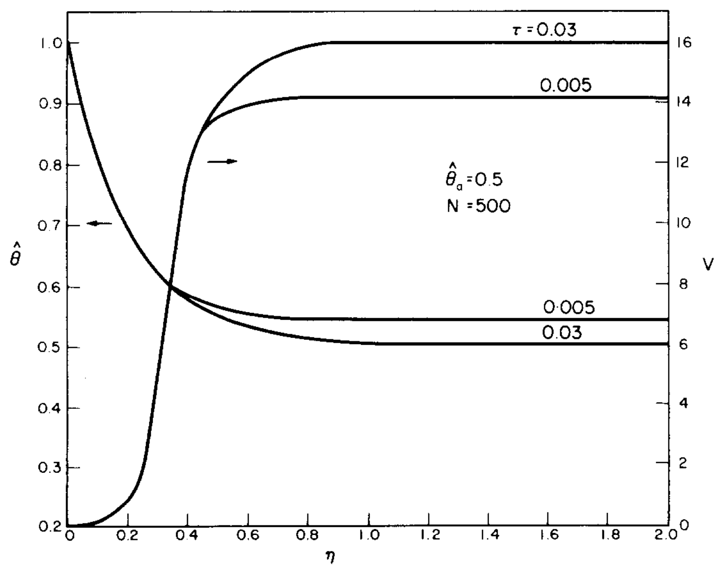

FIG. 6. Mean and variance of the temperature profile for $N = 500$ and $\hat{\theta}_a = 0.5$.

The heat removed from the channel flow to the surrounding medium may be determined using the results presented in this set of figures.

CONCLUDING REMARKS

The results confirm that over the range of conditions covered in this work, conventional solutions have a predictive accuracy for low values of the convective parameter. As the magnitude of the convective parameter increases, the thermal field needs to be described by both the mean and the variance. Moreover, for fixed coordinate and time in the channel, the variance increases as the convective parameter increases.

It is recommended that further work be done to examine the effects of other phenomena in the framework of random heat transfer.

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APPENDIX 1

Statistical properties of the ambient temperature

The properties of a random process $x(t)$ are determined by carrying out some experiments and applying the methods of data analysis [21]. The mean $\hat{x}(t_1)$ of $x(t)$ at some time t_1, and the covariance $v(t_1, t_2)$ at two times, t_1 and t_2, are computed by

$$\hat{x}(t_1) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} x_k(t_1) \quad (1\text{-}1)$$

and

$$v(t_1, t_2) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} x_k(t_1)x_k(t_2) - \hat{x}(t_1)\hat{x}(t_2) \quad (1\text{-}2)$$

where the RHS of equations (1-1) and (1-2) represent the ensemble averages of sample functions at fixed instants. When $x(t_1)$ and $v(t_1, t_2)$ do not depend upon the fixed instants, we may express $\hat{x}(t_1)$ and $v(t_1, t_2)$ as $\hat{x}$ and $v(t_1 - t_2)$, respectively. In this case, $x(t)$ is said to be stationary.

If $x(t)$ is stationary, and also, the statistics do not differ when computed over different sample functions, then $x(t)$ is said to be ergodic. For this case, we may determine the statistics by performing time averages over a single sample function. That is,

$$\hat{x} = \lim_{T\to\infty} \frac{1}{T} \int_0^T x_k(t)\,\mathrm{d}t \quad (1\text{-}3)$$

and

$$\begin{aligned} v(t_1 - t_2) &= v(\Delta) \\ &= \lim_{T\to\infty} \frac{1}{T} \int_0^T x_k(t)x_k(t+\Delta)\,\mathrm{d}t - \hat{x}^2 \end{aligned} \quad (1\text{-}4)$$

where T and Δ denote time and $t_1 - t_2$, respectively.

The covariance, denoted by equation (5), contains the Dirac's delta function and sometimes this is called a white process. Although we assume a white process for the ambient temperature, we can easily extend the procedure discussed here to more general classes because general processes (for example, the Markov process) are generated from a white process.

APPENDIX 2

Derivation of the covariance equation

The partial derivative of $v(\xi, \eta, \tau)$ with respect to time is given by

$$\begin{aligned} \frac{\partial v(\xi, \eta, \tau)}{\partial t} &= \frac{\partial}{\partial \tau} E[\tilde{\theta}(\xi, \tau)\tilde{\theta}(\eta, \tau) \\ &= E\left[\frac{\partial \tilde{\theta}(\xi, \tau)}{\partial \tau} \tilde{\theta}(\eta, \tau)\right] \\ &\quad + E\left[\tilde{\theta}(\xi, \tau) \frac{\partial \tilde{\theta}(\eta, \tau)}{\partial \tau}\right]. \end{aligned} \quad (2\text{-}1)$$

From equations (1) and (6), we have the differential equation for $\tilde{\theta}$ written as

$$\begin{aligned} \frac{\partial \tilde{\theta}(\eta, \tau)}{\partial \tau} &= \frac{\partial^2 \tilde{\theta}(\eta, \tau)}{\partial \eta^2} - U \frac{\partial \tilde{\theta}(\eta, \tau)}{\partial \eta} \\ &\quad - N[\tilde{\theta}(\eta, \tau) - \tilde{\theta}_a(\tau)]. \end{aligned} \quad (2\text{-}2)$$

Substituting equation (2-2) into the second term of the RHS of equation (2-1), yields:

$$\text{second term} = E\left\{\tilde{\theta}(\xi,\tau)\left[\frac{\partial^2\tilde{\theta}(\eta,\tau)}{\partial\eta^2} - U\frac{\partial\tilde{\theta}(\eta,\tau)}{\partial\eta} - N\tilde{\theta}(\eta,\tau) + N\tilde{\theta}_a(\tau)\right]\right\}$$

$$= \frac{\partial^2 v(\xi,\eta,\tau)}{\partial\eta^2} - U\frac{\partial v(\xi,\eta,\tau)}{\partial\eta} - Nv(\xi,\eta,\tau) + NE[\tilde{\theta}(\xi,\tau)\tilde{\theta}_a(\tau)] \quad (2\text{-}3)$$

where $\tilde{\theta}(\xi,\tau)$ is evaluated by [22]:

$$\tilde{\theta}(\xi,\tau) = \int_0^\tau d\tau' \int_0^\infty G(\xi,\xi',\tau-\tau')N\tilde{\theta}_a(\tau')d\tau'. \quad (2\text{-}4)$$

Here, the function G represents the temperature caused by an instantaneous source acting at an instant τ', and satisfying the relations

$$G(\xi,\xi',0) = \delta(\xi-\xi') \quad (2\text{-}5)$$

$$\frac{\partial G(\xi,\xi',\tau-\tau')}{\partial\tau} = \left(\frac{\partial^2}{\partial\xi^2} - U\frac{\partial}{\partial\xi} - N\right)G(\xi,\xi',\tau-\tau'). \quad (2\text{-}6)$$

Therefore,

$$E[\tilde{\theta}(\xi,\tau)\tilde{\theta}_a(\tau)] = \int_0^\tau d\tau' \int_0^\infty G(\xi,\xi',\tau-\tau')N^2E[\tilde{\theta}_a(\tau)\tilde{\theta}_a(\tau')]d\xi'$$

$$= \int_0^\tau d\tau' \int_0^\infty G(\xi,\xi',\tau-\tau')N^2W\delta(\tau-\tau')d\xi'$$

$$= \frac{N^2W}{2}. \quad (2\text{-}7)$$

Ultimately, the first term of the RHS of equation (2-1) will be evaluated in the same manner. Consequently, equation (2-1) becomes equation (25).

TRANSFERT ERRATIQUE DE CHALEUR DANS UN CANAL PLAT AVEC DES VARIATIONS DANS LE TEMPS DE TEMPERATURE AMBIANTE

Résumé—Cet article présente une étude analytique du transfert erratique de chaleur à l'entrée, d'un fluide entre deux plans parallèles. L'environnement est sujet à des oscillations de température variant au hasard. L'analyse utilisée est centrée sur une formulation localisée dans la direction transversale. Des résultats du calcul fournissent les paramètres statistiques tels que la distribution de température moyenne et la distribution en variance pour un groupe de situations. Des estimations de l'état erratique de la température du fluid peuvent être déduites d'un ensemble de configurations qui concernent plusiers exemples.

WÄRMEÜBERGANG IN FLACHEN KANÄLEN BEI ZEITLICH STOCHASTISCHEM VERLAUF DER UMGEBUNGSTEMPERATUR

Zusammenfassung—Diese Arbeit behandelt eine analytische Untersuchung des stochastischen Wärmeübergangs eines Fluids im Eintrittsbereich eines Rechteckkanals. Die Umgebung des Kanals erfährt willkürlich veränderliche Temperaturschwankungen. Die angewandte mathematische Methode basiert auf einem Knotenmodell in Querrichtung des Kanals. Die errechneten Ergebnisse liefern die statistischen Parameter wie die mittlere Temperaturverteilung und die Verteilung der Varianz für eine Gruppe von Wärmeübergangssituationen. Annahmen über das Zufallsverhalten der Fluidtemperatur können mit Hilfe einiger Abbildungen, die mehrere Beispiele enthalten, vorgenommen werden.

ТЕПЛООБМЕН В ПЛОСКИХ КАНАЛАХ ПРИ СЛУЧАЙНОМ ИЗМЕНЕНИИ ВО ВРЕМЕНИ ТЕМПЕРАТУРЫ ОКРУЖАЮЩЕЙ СРЕДЫ

Аннотация — В статье рассматривается аналитическое исследование случайного теплообмена жидкости во входной области плоскопараллельного канала. Окружающий канал среда испытывает случайные колебания температуры. Даётся общая модель процесса передачи тепла по сечению канала. На основании расчётных значений для нескольких рассматриваемых случаев получены такие статистические параметры, как среднее распределение температур и дисперсионное распределение. Случайный характер изменения температуры можно проследить на ряде рисунков.